Network Localization from Relative Bearing Measurements

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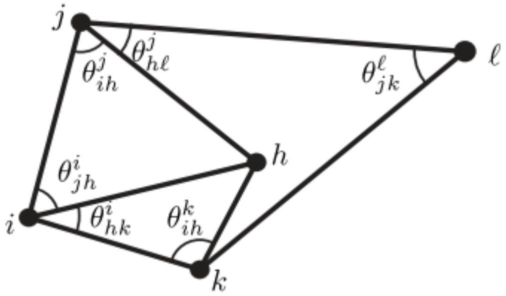
1) Problem setup



- Nodes measure ***relative angles*** to other nodes
- ***No global reference frame***
- e.g., camera networks
- **Problem:** What is the layout of the network?
  - Up to *translation*, *scaling*, *rotation*
- For now, consider the problem in 2D
  - Locations given by complex numbers

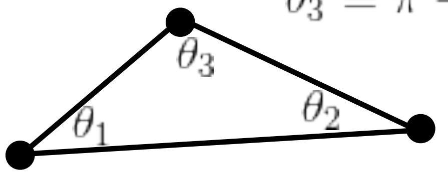
4) Optimization

1. Alternating optimization:
   - Solve for r, solve for x, ...
   - Each can be solved optimially in terms of the other
2. Eigenvalue minimization:
   - Minimize 2nd-smallest eigenvalue of $M(r)$

Alternating optimization is simplest and works best in practice

2) Triangular constraints

$$\theta_3 = \pi - \theta_1 - \theta_2$$



- Three nodes that see each other
- =>Three angle constraints (**two** independent)
  - Solution: any triangle with those angles
- Let $t = \begin{bmatrix} t_1 & t_2 & t_3 \end{bmatrix}^T$ be one such triangle
- **The set of all triangles with the same internal angles can be written as**

$$x = \alpha t + \beta \mathbb{1}$$

scaling/rotation

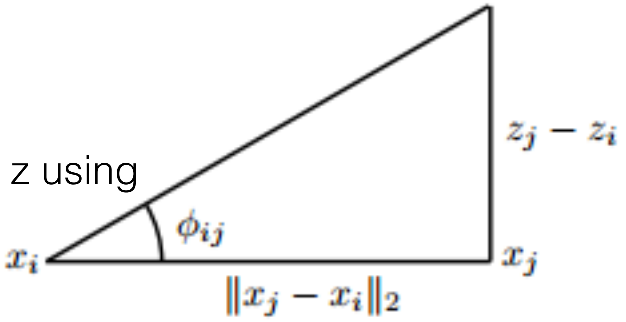
translation

for complex numbers α **and** β

- Equivalently, is a **linear constraint:**
  - $x \in \text{span}\{t, \mathbb{1}\}$
  - $x \perp \overline{t \times \mathbb{1}}$
  - $t^*_\perp x = 0, \quad \text{where } t_\perp = \overline{t \times \mathbb{1}}$
- For a network of triangles, solve $Ax = 0$

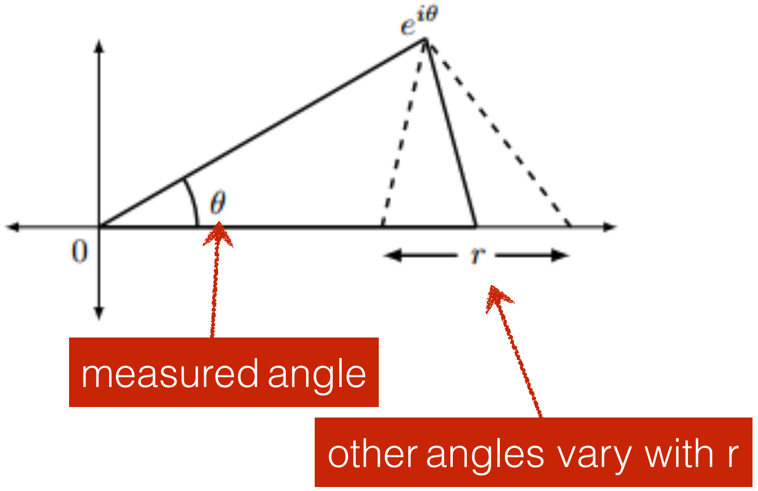
5) Extension to 3D with gravity

- Assume a **veritical axis is known**
  - Robot may have accelerometer, or camera can estimate vertical from buildings.
- If vertical axis is known:
  1. Project to x-y plane and solve
  2. Fix x-y positions and estimate z using a linear system of equations:



3) Extension to general constraints

- One angle measurement is **a triangle with a missing angle**



measured angle

other angles vary with r

- One angle measurement can be written as a **parameterized** linear constraint:

$$A(r)x = 0$$

  - Unknowns are x, r
- Optimization problem:

$$\min \|A(r)x\|_2^2 = x^* M(r) x$$
$$\text{subject to } x^* \mathbf{1} = 0$$

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Random networks

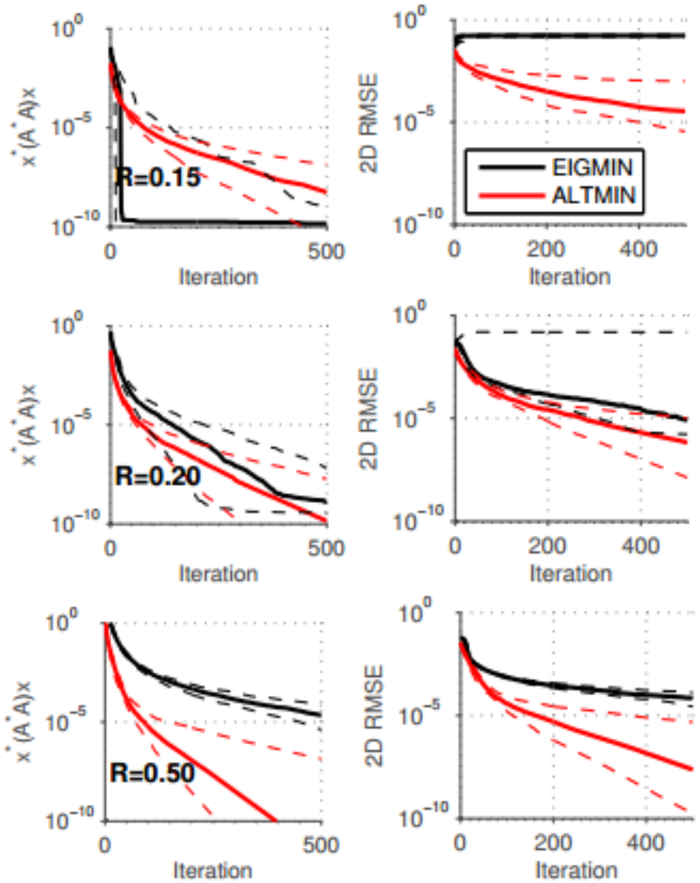
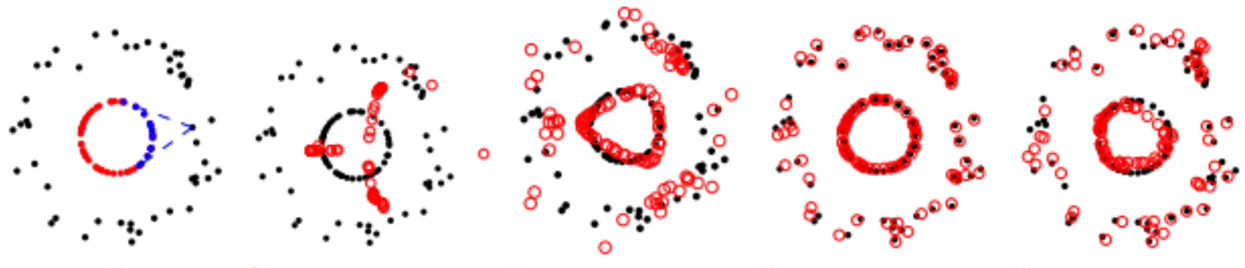


Fig. 4: Comparison of EIGMIN and ALTMIN algorithms on 100 random networks of 100 points where each node has a sensor range of a fixed radius R. We plot the median of all runs, with the 25^{th} and 75^{th} percentiles plotted as dashed lines. **Top row:** $R = 0.15$, **Middle row:** $R = 0.2$, **Bottom row**: $R = 0.5$.

Structure from Motion (2D, synthetic data)



(a)

(b)
$\lambda = 0$
3.64×10^{-4}
0.8558

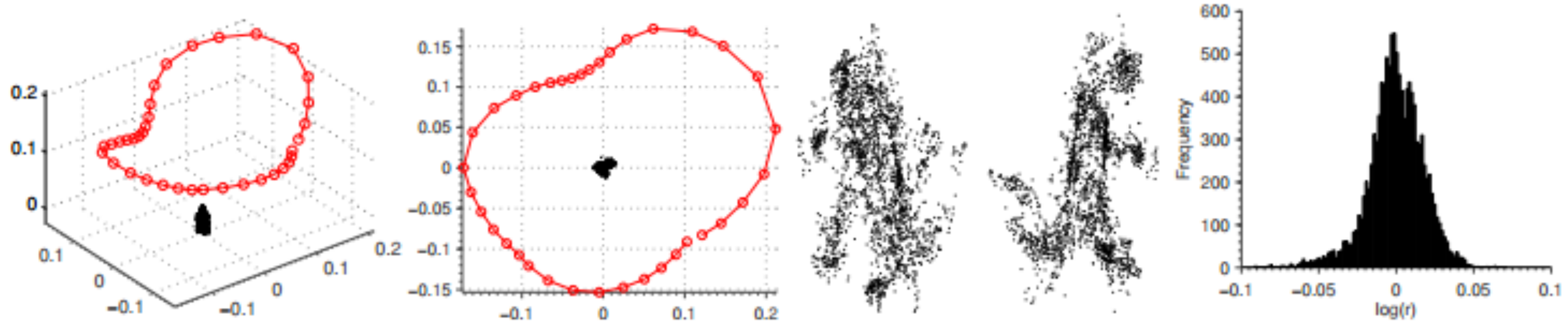
(c)
$\lambda = 10^{-6}$
1.65×10^{-3}
0.3437

(d)
$\lambda = 10^{-4}$
1.96×10^{-3}
0.0714

(e)
$\lambda = 10^{-2}$
6.00×10^{-2}
0.1705

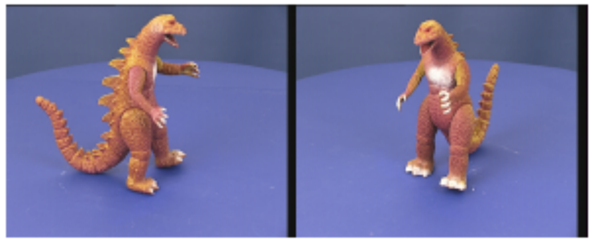
Fig. 5: **Synthetic 2D SFM dataset** **(a)** A circular object with 50 visible points is surrounded by 50 cameras. Each camera is able to see points on the side of the circle within its field of view, as depicted here in blue. **(b)-(e)** Reconstruction using ALTMIN (red circles) using various values of λ for regularization. The numbers shown are the value of λ (top), the matrix cost $x^*(A^*A)x$ (middle), and the 2D RMSE (bottom). The best reconstruction is given for $\lambda \approx 10^{-4}$.

Structure from Motion (3D, real data)



(a) (b) (c) (d) (e)

Fig. 6: **Results on Dinosaur dataset** **(a),(b)** Estimated 3D layout of all cameras (red) and scene points (black). **(c),(d)** Two views of the estimated 3D Dinosaur model. **(e)** Histogram of the log of radius values for the given reconstruction.



(a) Frame 5 (b) Frame 10

Fig. 8: Two frames from the Dinosaur dataset used in our experiments.